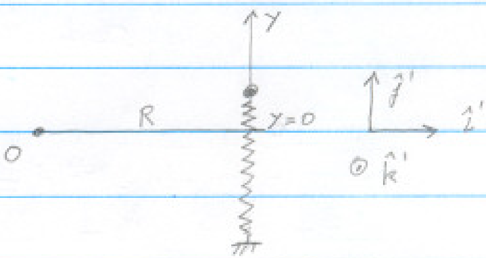
EXAMEN DE MECÁNICA CLÁSICA, 14/7/2001

- SOLUCIÓN -

Ej 1



$S'\{O, \hat{\imath}', \hat{j}'\}$ FIJO AL DISCO

$$\vec{a}_T = \vec{\omega}\wedge(\vec{\omega}\wedge\vec{r}\,') = \omega^2\hat{k}\wedge[\hat{k}\wedge(R\hat{\imath}' + y\hat{j}')]$$
$$= -\omega^2(R\hat{\imath}' + y\hat{j}')$$

$$\vec{a}_C = 2(\omega\hat{k})\wedge(\dot{y}\hat{j}') = -2\omega\dot{y}\hat{\imath}'$$

a) $\hat{\imath}')\quad 0 = N + 2m\omega\dot{y} + m\omega^2 R$

$\hat{j}')\quad m\ddot{y} = -ky + m\omega^2 y \longrightarrow \boxed{\ddot{y} = \left(\omega^2 - \frac{k}{m}\right)y}$

b) i) $\ddot{y} = -\left(\frac{k}{m} - \omega^2\right)y$ OSCILADOR $\Longleftrightarrow \boxed{\Omega^2 = \frac{k}{m} - \omega^2 > 0}$

ii) $\ddot{y} = -\Omega^2 y \Rightarrow y = A\cos\Omega t + B\,\mathrm{sen}\,\Omega t$ $\quad\begin{cases}\dot{y}(0) = \Omega B = 0 \Rightarrow B = 0\\ y(0) = A = -\ell\end{cases}$

$\Rightarrow \boxed{y(t) = -\ell\cos(\Omega t)}$

c) $\vec{N} = -(2m\omega\dot{y} + m\omega^2 R)\hat{\imath}'$

$\vec{v} = \vec{v}\,' + \vec{v}_{oo'} + \vec{\omega}\wedge\vec{r}\,' = \dot{y}\hat{j}' + \omega\hat{k}\wedge(R\hat{\imath}' + y\hat{j}') = -\omega y\hat{\imath}' + (\dot{y} + \omega R)\hat{j}'$

i)

$\Rightarrow P = \vec{N}\cdot\vec{v} = 2m\omega^2 y\dot{y} + m\omega^3 R y \qquad \dot{y} = \Omega\ell\,\mathrm{sen}\,\Omega t$

$\Rightarrow P = -2m\omega^2\Omega\ell^2\,\mathrm{sen}(\Omega t)\cos(\Omega t) - m\omega^3 R\ell\cos(\Omega t)$

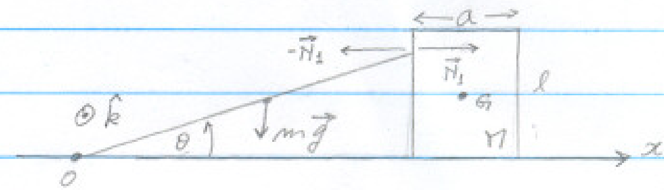
ii) $W = \int_0^{T/4} P(t)\,dt = 2m\omega^2\ell^2\int_1^0 \cos(\Omega t)\,d[\cos(\Omega t)] - m\omega^3 R\ell\int_0^{T/4}\cos(\Omega t)\,dt$

$= 2m\omega^2\ell^2\left(\frac{0}{2} - \frac{1}{2}\right) - m\frac{\omega^3\ell R}{\Omega}\left[\mathrm{sen}(\pi/2) - 0\right]$

$= -m\omega^2\ell^2 - m\frac{\omega^3}{\Omega}R\ell$

Ej 2

a)



$$x_G = 2\ell\cos\theta + \frac{a}{2}$$

$\Rightarrow$

$$\dot{x}_G = -2\ell\,\dot{\theta}\,\text{sen}\,\theta$$

$$\ddot{x}_G = -2\ell\,(\ddot{\theta}\,\text{sen}\,\theta + \dot{\theta}^2\cos\theta)$$

b) 2^{da} CARDINAL EN O: $\quad (\hat{k})\ \ I_O\,\ddot{\theta} = -mg\ell\cos\theta + 2\ell N_1\,\text{sen}\,\theta$

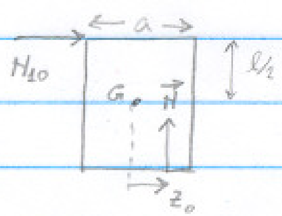
$$\left.\begin{array}{l} I_O = \frac{1}{3}m\ell^2 + m\ell^2 = \frac{4}{3}m\ell^2 \\ N_1 = M\ddot{x}_G = -\frac{4}{3}m\ell(\ddot{\theta}\,\text{sen}\,\theta + \dot{\theta}^2\cos\theta) \end{array}\right\} \Rightarrow$$

$$\frac{4}{3}m\ell^2\ddot{\theta} = -mg\ell\cos\theta - \frac{8}{3}m\ell^2(\ddot{\theta}\,\text{sen}\,\theta + \dot{\theta}^2\cos\theta)\,\text{sen}\,\theta$$

$$\Rightarrow \boxed{(1 + 2\,\text{sen}^2\theta)\,\ddot{\theta} = -2\dot{\theta}^2\,\text{sen}\,\theta\cos\theta - \frac{3g}{4\ell}\cos\theta}$$

c)

$t = 0$



$$(\hat{k})\ -N_{10}\frac{\ell}{2} + N z_0 = 0 \quad \Rightarrow \quad z_0 = \frac{N_{10}\,\ell}{2N} = N_{10}\frac{3\ell}{4mg} \quad (>0)$$

$$N = \left(\frac{2}{3}m\right)g$$

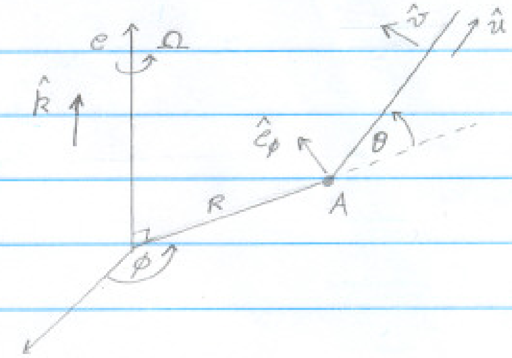
DEBE SER $\quad z_0 > -a/2$ ✓ $\quad$ y $\quad z_0 < a/2$

$$\theta_0 = 30° \ ; \ \dot{\theta}_0 = 0 \quad \Rightarrow \quad \left(1 + 2\cdot\left(\tfrac{1}{2}\right)^2\right)\ddot{\theta}_0 = -\frac{3g}{4\ell}\frac{\sqrt{3}}{2} \quad \Rightarrow \quad \ddot{\theta}_0 = -\frac{\sqrt{3}}{4}\frac{g}{\ell}$$

$$N_{10} = \frac{2}{3}m\,\ddot{x}_G(0) = \frac{2}{3}m\left(-2\ell\,\ddot{\theta}_0\,\frac{1}{2}\right) = \frac{2}{3}m\left[-\ell\left(-\frac{\sqrt{3}}{4}\frac{g}{\ell}\right)\right] = \frac{1}{2\sqrt{3}}mg$$

$$\Rightarrow z_0 = \frac{1}{2\sqrt{3}}mg\,\frac{3\ell}{4mg} = \frac{\sqrt{3}}{8}\ell < a/2 \quad \Leftrightarrow \quad \boxed{\sqrt{3}\,\ell < 4a}$$

Ej 3



$\{\hat{u}, \hat{v}, \hat{e}_\phi\}$ SOLIDARIA

a) $\vec{\omega} = \Omega \hat{k} - \dot{\theta}\,\hat{e}_\phi$

$= \Omega \operatorname{sen}\theta\, \hat{u} + \Omega \cos\theta\, \hat{v} - \dot{\theta}\,\hat{e}_\phi$

b)

EN $\{\hat{u}, \hat{v}, \hat{e}_\phi\}$

$$\mathbb{I}_A = \frac{1}{3} m R^2 \begin{bmatrix} 0 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} + m R^2 \begin{bmatrix} 0 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} = \frac{4}{3} m R^2 \begin{bmatrix} 0 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$\vec{L}_A = \mathbb{I}_A \vec{\omega} + m (G - A) \wedge \vec{v}_A \quad // \quad \mathbb{I}_A \vec{\omega} = \frac{4}{3} m R^2 \left(\Omega \cos\theta\, \hat{v} - \dot{\theta}\,\hat{e}_\phi \right)$

$m (G - A) \wedge \vec{v}_A = m (R\hat{u}) \wedge (R\Omega)\,\hat{e}_\phi = m R^2 \Omega\, \hat{v}$

$\Rightarrow \vec{L}_A = m R^2 \Omega \left(\frac{4}{3}\cos\theta + 1 \right) \hat{v} - \frac{4}{3} m R^2 \dot{\theta}\,\hat{e}_\phi$

$\frac{d\hat{v}}{dt} = \vec{\omega} \wedge \hat{v} = -\Omega \operatorname{sen}\theta\, \hat{e}_\phi - \dot{\theta}\,\hat{u} \qquad \frac{d\hat{e}_\phi}{dt} = \vec{\omega} \wedge \hat{e}_\phi = \Omega \operatorname{sen}\theta\, \hat{v} - \Omega \cos\theta\, \hat{u}$

$\Rightarrow \dot{\vec{L}}_A \cdot \hat{e}_\phi = -m R^2 \Omega^2 \left(\frac{4}{3}\cos\theta + 1 \right) \operatorname{sen}\theta - \frac{4}{3} m R^2 \ddot{\theta}$

2^{da} CARDINAL EN A: $\dot{\vec{L}}_A = \vec{M}_A^{ext} + \vec{P} \wedge \vec{v}_A$

$\vec{v}_A = R\Omega\, \hat{e}_\phi$

$\vec{v}_G = \vec{v}_A + \vec{\omega} \wedge (G - A) = R\Omega\, \hat{e}_\phi + \vec{\omega} \wedge (R\hat{u}) = R\dot{\theta}\, \hat{v} + R\Omega (1 + \cos\theta)\, \hat{e}_\phi$

$\Rightarrow \vec{P} \wedge \vec{v}_A = m \vec{v}_G \wedge \vec{v}_A = -m R^2 \Omega \dot{\theta}\, \hat{u}$

$\therefore \ \hat{e}_\phi)\ -m R^2 \Omega^2 \left(\frac{4}{3}\cos\theta + 1 \right) \operatorname{sen}\theta - \frac{4}{3} m R^2 \ddot{\theta} = m g R \cos\theta$

$$\Rightarrow \boxed{\ddot{\theta} = -\Omega^2 \operatorname{sen}\theta \cos\theta - \frac{3}{4}\Omega^2 \operatorname{sen}\theta - \frac{3}{4}\frac{g}{R}\cos\theta}$$

c) $\theta_0 = -45°$ $\Rightarrow$ DEBE SER: $\ddot{\theta}_0 = 0$

$$\Rightarrow 0 = -\Omega^2 \left(-\frac{1}{\sqrt{2}}\right)\frac{1}{\sqrt{2}} - \frac{3}{4}\Omega^2\left(-\frac{1}{\sqrt{2}}\right) - \frac{3g}{4R}\frac{1}{\sqrt{2}} = \Omega^2\left(\frac{1}{2} + \frac{3}{4\sqrt{2}}\right) - \frac{3g}{4\sqrt{2}R}$$

$$\Omega^2\left(\frac{2\sqrt{2}}{3} + 1\right) = \frac{g}{R} \qquad \Rightarrow \boxed{\frac{g}{\Omega^2 R} = 1 + \frac{2\sqrt{2}}{3}}$$