MECÁNICA CLÁSICA 2012

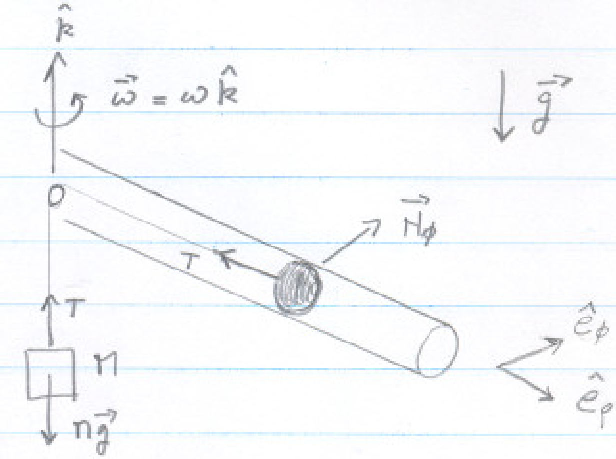
PRIMER PARCIAL – SOLUCIÓN

Ej 1

a

$S'\{O, \hat{e}_\rho, \hat{e}_\phi, \hat{k}\}$:

$$\begin{cases} \vec{a}_T = \vec{\omega} \wedge (\vec{\omega} \wedge \vec{r}\,') = -\omega^2 r \,\hat{e}_\rho \\ \vec{a}_c = 2\vec{\omega} \wedge \vec{v}\,' = 2\omega \dot{r}\,\hat{e}_\phi \end{cases}$$



m) $\hat{e}_\rho$) $m\ddot{r} = m\omega^2 r - T$

$\hat{e}_\phi$) $0 = N_\phi - 2m\omega\dot{r}$

M) $M\ddot{y} = T - Mg$

$$\ddot{y} = \ddot{r} \Rightarrow m\ddot{r} = m\omega^2 r - M\ddot{r} - Mg \Rightarrow \ddot{r} = \frac{m\omega^2}{M+m} r - \frac{Mg}{M+m}$$

b

$r(0) = L$

$\dot{r}(0) = 0$ DEBE SER $\ddot{r}(0) = \frac{m\omega^2}{M+m} L - \frac{Mg}{M+m} < 0 \Rightarrow m\omega^2 L < Mg$

c

SEA $\begin{cases} \Omega^2 = \frac{m\omega^2}{M+m} \\ \alpha = \frac{Mg}{M+m} \end{cases} \longrightarrow \ddot{r} = \Omega^2 r - \alpha$

$$\Rightarrow r(t) = r_{GH}(t) + r_{PNH}(t)$$
$$= A \,\text{Senh}\, \Omega t + B \cosh \Omega t + \frac{\alpha}{\Omega^2}$$

CI: $\dot{r}(0) = \Omega A = 0$

$r(0) = B + \frac{\alpha}{\Omega^2} = L \longrightarrow r(t) = \underbrace{\left(L - \frac{\alpha}{\Omega^2}\right)}_{<0} \cosh \Omega t + \frac{\alpha}{\Omega^2}$

d

S: $W^{EXT} = W_{N_\phi}$ $\begin{matrix} \vec{N}_\phi = 2m\omega\dot{r}\,\hat{e}_\phi \\ \vec{v} = \dot{r}\,\hat{e}_\rho + r\omega\,\hat{e}_\phi \end{matrix} \Bigg\} \Rightarrow P_{N_\phi} = 2m\omega^2 r\dot{r}$

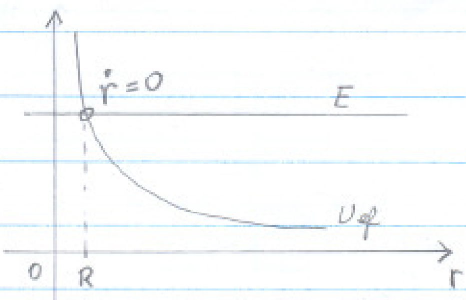
$$W^{EXT} = \int_0^{t_f} 2m\omega^2 r\dot{r}\,dt = \int_L^0 2m\omega^2 r\,dr = -m\omega^2 L^2 \quad (< 0 \checkmark)$$

Ej 2

a) $\vec{F} = -\frac{\lambda}{r^3}\hat{e}_r \Rightarrow U(r) = -\frac{\lambda}{2r^2}$

$$U_{ef}(r) = \frac{\ell^2}{2mr^2} + U(r) = \frac{\ell^2}{2mr^2} - \frac{\lambda}{2r^2} = \frac{\ell^2 - m\lambda}{2mr^2}$$

i $\ell^2 > m\lambda$



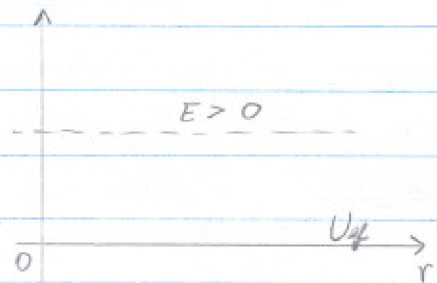
- $E > 0$

TIENE 1 PUNTO DE RETROCESO, SE MUEVE EN $R \leq r < \infty$

- $E \leq 0$

NO ES POSIBLE EL MOVIMIENTO

ii $\ell^2 = m\lambda$



$$E = \frac{1}{2}m\dot{r}^2 + U_{ef}(r) = \frac{1}{2}m\dot{r}^2$$

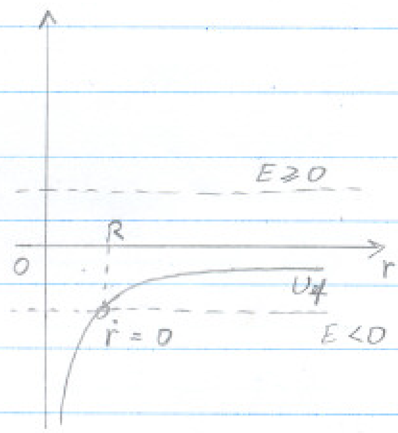
- $E > 0 \Rightarrow \dot{r} \neq 0$

r AUMENTA O DISMINUYE A RITMO CTE.

- $E = 0 \Rightarrow \dot{r} = 0$

MOV. CIRCULAR

iii $\ell^2 < m\lambda$



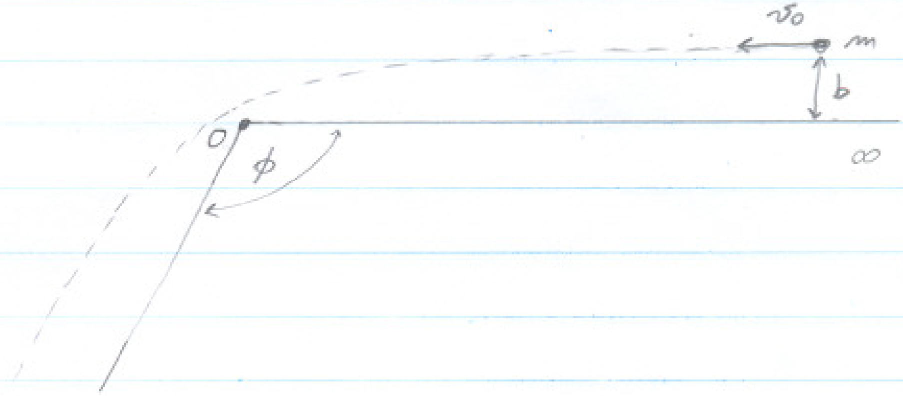
- $E \geq 0$

MOVIMIENTO EN $0 < r < \infty$ (CAE HACIA O Ó ESCAPA)

- $E < 0$

MOVIMIENTO ACOTADO EN $0 \leq r \leq R$ (CAE HACIA 0).

b



$$u = \frac{1}{r} \Rightarrow u'' + u = -\frac{m}{\ell^2}\frac{F(u)}{u^2}$$

$$F(u) = -\lambda u^3$$

$\Rightarrow$

$$u'' + u = +\frac{m\lambda}{\ell^2} u$$

$$u'' = -\left(1 - \frac{m\lambda}{\ell^2}\right) u \qquad \ell^2 > m\lambda \Rightarrow \Omega^2 = 1 - \frac{m\lambda}{\ell^2} > 0$$

$$u'' = -\Omega^2 u \longrightarrow u(\theta) = A\,\mathrm{Sen}(\Omega\theta) + B\cos(\Omega\theta)$$

C. I. $\left|\begin{array}{l} r(0) = \infty \\ \dot r(0) = -v_0 \end{array}\right. \longrightarrow \left|\begin{array}{l} u(\theta = 0) = 0 = B \\ u'(\theta = 0) = -\dfrac{m}{\ell}\dot r(0) = \dfrac{m v_0}{\ell} = \Omega A \end{array}\right.$

$$\Rightarrow u(\theta) = \frac{m v_0}{\Omega \ell}\,\mathrm{Sen}(\Omega\theta)$$

$$r(\theta) = \frac{\Omega \ell}{m v_0}\frac{1}{\mathrm{Sen}(\Omega\theta)}$$

ASÍNTOTAS $\quad r = \infty \longleftrightarrow u = 0 \Rightarrow \mathrm{Sen}(\Omega\theta) = 0 \longrightarrow \theta = 0$ (INICIAL)

$\theta = \dfrac{\pi}{\Omega}$ (AL VOLVER A ∞)

EL ÁNGULO DE DESVIACIÓN ES $\phi = 2\pi - \dfrac{\pi}{\Omega}$