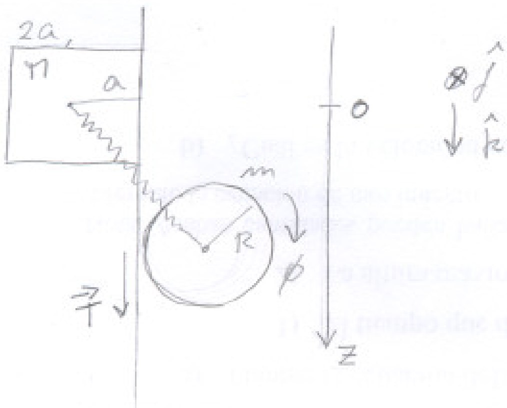
SEGUNDO PARCIAL DE MECÁNICA CLÁSICA 2010 - SOLUCIONES

EJ 1



<u>a</u>

DISCO: $m\vec{a} = mg\hat{k} + T\hat{k} + \vec{F}_R$

$$\vec{F}_R = -k\vec{l} \Rightarrow \vec{F}_R \cdot \hat{k} = -kz$$

$$\Rightarrow m\ddot{z} = mg + T - kz$$

2° CARD

$$\hat{j})\quad I\ddot{\phi} = -RT = -\left(\tfrac{1}{2}mR^2\right)\frac{\ddot{z}}{R} \Rightarrow T = -\frac{1}{2}m\ddot{z}$$

$$\Rightarrow m\ddot{z} = mg - \frac{1}{2}m\ddot{z} - kz \Rightarrow \boxed{\ddot{z} = -\frac{2k}{3m}z + \frac{2}{3}g}$$

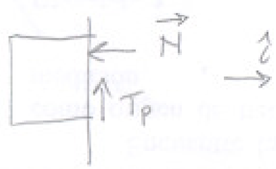
SEA $\omega_0^2 = \frac{2k}{3m}$

SGH: $z_H = A\,\mathrm{sen}(\omega_0 t) + B\cos(\omega_0 t)$

SPHH: $z_P = \frac{mg}{k}$ (cte)

$$\Rightarrow \left.\begin{array}{l} z(0) = B + \frac{mg}{k} = 0 \\ \dot{z}(0) = \omega_0 A = 0 \end{array}\right\} \Rightarrow \boxed{z(t) = \frac{mg}{k}\left[1 - \cos(\omega_0 t)\right] \geq 0 \quad \forall t}$$

<u>b</u>

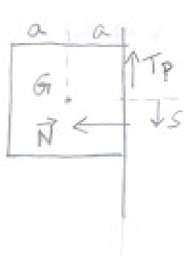


$$\hat{i})\quad -N + \vec{F}_R \cdot \hat{i} = -N + k\vec{l} \cdot \hat{i} = -N + k(a+R) = 0$$

$$\hat{j})\quad Mg - T_P + kz = 0$$

$$\Rightarrow \boxed{\begin{array}{l} N = k(a+R) \\ T_P = Mg + kz \end{array}}$$

<u>c</u>



2° CARD EN G:

$$\hat{j})\quad sN - aT_p = 0$$

$$s = \frac{T_P}{N}a = \frac{Mg + kz}{k(a+R)}a$$

NO VUELCO: $-a \leq s \leq a$

$$-a \leq \frac{mg + kz}{k(a+R)} a \Rightarrow -k(a+R) - mg \leq kz \quad \forall z$$

SIEMPRE SE CUMPLE, PORQUE $z \geq 0 \ \forall t$. (LA PLACA NO VA A VOLCAR RESPECTO AL VÉRTICE SUPERIOR).

$$\frac{mg + kz}{k(a+R)} a \leq a \Rightarrow kz \leq k(a+R) - mg$$

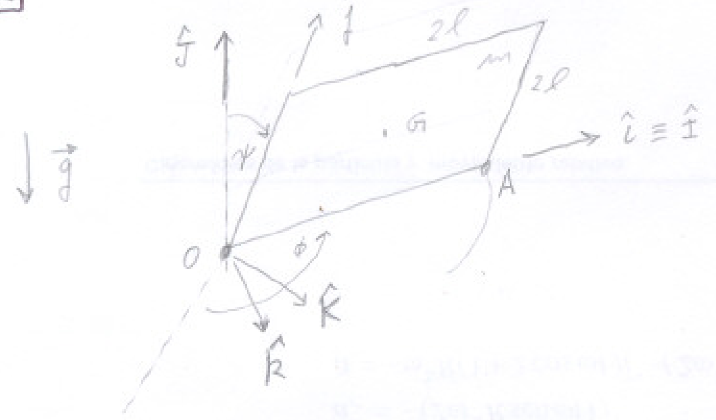
$$z_{min} = 0$$

$$z_{max} = \frac{2mg}{k}$$

$\Rightarrow$ 1) $\boxed{k(a+R) > mg}$

2) $k \cdot \frac{2mg}{k} \leq k(a+R) - mg \Rightarrow \boxed{(2m+m)g \leq k(a+R)}$

EJ 2



$$\vec{\omega} = \dot{\phi}\hat{J} + \dot{\psi}\hat{\imath} = \Omega\hat{J} + \dot{\psi}\hat{\imath}$$

$$\hat{J} = \cos\psi\,\hat{j} - \operatorname{sen}\psi\,\hat{k}$$

$$\vec{\omega} = \dot{\psi}\hat{\imath} + \Omega\cos\psi\,\hat{j} - \Omega\operatorname{sen}\psi\,\hat{k}$$

$\{G, \hat{\imath}, \hat{j}, \hat{k}\}$

$$\mathbb{I}_G = \frac{1}{3} m l^2 \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 2 \end{bmatrix} \Rightarrow \vec{L}_G = \frac{1}{3} m l^2 \left[\dot{\psi}\hat{\imath} + \Omega\cos\psi\,\hat{j} - 2\Omega\operatorname{sen}\psi\,\hat{k}\right]$$

$$\vec{L}_O = \vec{L}_G + (G-O)\wedge\vec{p} \qquad G-O = l\hat{\imath} + l\hat{j}$$

$$\vec{v}_G = \vec{\omega}\wedge(G-O) = l\dot{\psi}\hat{k} - l\Omega\cos\psi\,\hat{k} - l\Omega\operatorname{sen}\psi\,\hat{j} + l\Omega\operatorname{sen}\psi\,\hat{\imath}$$

$$= l(\dot{\psi} - \Omega\cos\psi)\hat{k} - l\Omega\operatorname{sen}\psi\,\hat{j} + l\Omega\operatorname{sen}\psi\,\hat{\imath}$$

$$(G-O)\wedge(m\vec{v}_G) = m l^2\left[-(\dot{\psi} - \Omega\cos\psi)\hat{j} - \Omega\operatorname{sen}\psi\,\hat{k} + (\dot{\psi} - \Omega\cos\psi)\hat{\imath} - \Omega\operatorname{sen}\psi\,\hat{k}\right]$$

$$= m l^2\left[(\dot{\psi} - \Omega\cos\psi)\hat{\imath} - (\dot{\psi} - \Omega\cos\psi)\hat{j} - 2\Omega\operatorname{sen}\psi\,\hat{k}\right]$$

$$\vec{L}_O = m l^2\left[\left(\frac{4}{3}\dot{\psi} - \Omega\cos\psi\right)\hat{\imath} - \left(\dot{\psi} - \frac{4}{3}\Omega\cos\psi\right)\hat{j} - \frac{8}{3}\Omega\operatorname{sen}\psi\,\hat{k}\right]$$

$$\hat{j} = \cos\psi\,\hat{J} + \operatorname{sen}\psi\,\hat{K} \qquad \hat{k} = -\operatorname{sen}\psi\,\hat{J} + \cos\psi\,\hat{K}$$

$$\vec{L}_O = m l^2\left[\left(\frac{4}{3}\dot{\psi} - \Omega\cos\psi\right)\hat{I} + \left(\frac{8}{3}\Omega\operatorname{sen}^2\psi + \frac{4}{3}\Omega\cos^2\psi - \dot{\psi}\cos\psi\right)\hat{J}\right.$$

$$\left. + \left(-\dot{\psi}\operatorname{sen}\psi + \frac{4}{3}\Omega\cos\psi\operatorname{sen}\psi - \frac{8}{3}\Omega\operatorname{sen}\psi\cos\psi\right)\hat{K}\right]$$

$$\vec{L}_O = m l^2\left[\left(\frac{4}{3}\dot{\psi} - \Omega\cos\psi\right)\hat{I} + \left(\frac{4}{3}\Omega(\operatorname{sen}^2\psi + 1) - \dot{\psi}\cos\psi\right)\hat{J}\right.$$

$$\left. - \left(\dot{\psi}\operatorname{sen}\psi + \frac{4}{3}\Omega\operatorname{sen}\psi\cos\psi\right)\hat{K}\right]$$

$$\dot{\vec{L}}_o \cdot \hat{K}\Big|_0 = ml^2\left[-\left(\frac{4}{3}\dot{\psi} - \Omega\cos\psi\right)\Omega - \left(\ddot{\psi}\,\mathrm{sen}\,\psi + \dot{\psi}^2\cos\psi + \frac{4}{3}\Omega\dot{\psi}\cos(2\psi)\right)\right]\Big|_0$$

$$= ml^2\left(\cancel{\Omega^2} - \frac{4}{3}\Omega^2 - \cancel{\Omega^2} - \frac{4}{3}\Omega^2\right) = -\frac{8}{3}ml^2\Omega^2$$

$$\psi_0 = 0$$

$$\dot{\psi}_0 = \Omega$$

$$= 2lN_0 - lmg$$

$$\Rightarrow N_0 = \frac{1}{2}mg - \frac{4}{3}ml\Omega^2 > 0 \Rightarrow \boxed{\Omega^2 < \frac{3}{8}\frac{g}{l}}$$